A PRESSURE-STABILIZED LAGRANGE-GALERKIN FINITE
ELEMENT SCHEME FOR AN OSEEN-TYPE DIFFUSIVE PETERLIN
MODEL

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Let Ω be a bounded domain in \( \mathbb{R}^d \) \( (d = 2) \) and \( T \) be a positive number. We consider an Oseen-type diffusive Peterlin system which describes a motion of an incompressible viscoelastic fluid,

\[
\begin{align*}
\frac{D_u}{Dt} - \nu \Delta u + \nabla p - \nabla (\text{tr} C) &= f & \text{in } \Omega \times (0,T), \\
\nabla \cdot u &= 0 & \text{in } \Omega \times (0,T), \\
\frac{D_C}{Dt} - \epsilon \Delta C - \left\{ (\nabla u) C + C (\nabla u)^T \right\} + (\text{tr} C)^2 C - (\text{tr} C) I &= F & \text{in } \Omega \times (0,T), \\
u &= 0, \quad C = 0 & \text{on } \partial \Omega \times (0,T), \\
u = u^0, \quad C = C^0 & \text{in } \Omega \at t = 0,
\end{align*}
\]

where \( u : \Omega \times (0,T) \rightarrow \mathbb{R}^d \) is the velocity, \( p : \Omega \times (0,T) \rightarrow \mathbb{R} \) is the pressure, \( C : \Omega \times (0,T) \rightarrow \mathbb{R}^{d \times d} \) is the conformation tensor, \( \frac{D_u}{Dt} = \frac{\partial}{\partial t} + w \cdot \nabla \), \( w : \Omega \times (0,T) \rightarrow \mathbb{R}^d \) is a given velocity, \( \nu \) and \( \epsilon \) are positive constants, \( f : \Omega \times (0,T) \rightarrow \mathbb{R}^d \), \( F : \Omega \times (0,T) \rightarrow \mathbb{R}^{d \times d} \), \( u^0 : \Omega \rightarrow \mathbb{R}^d \) and \( C^0 : \Omega \rightarrow \mathbb{R}^{d \times d} \) are given functions, \( \text{tr} C \) means the trace of \( C \). For this problem we present a Lagrange-Galerkin finite element scheme with Brezzi-Pitkaranta type pressure stabilization, where all unknown functions \( (u, p, C) \) are approximated by \( P1 \) elements, and show that
the finite element solution \((u_h, p_h, C_h)\) converges to the exact solution \((u, p, C)\) in order \(\Delta t + h\), where \(\Delta t\) is the time increment and \(h\) is the representative element size.

REFERENCES
